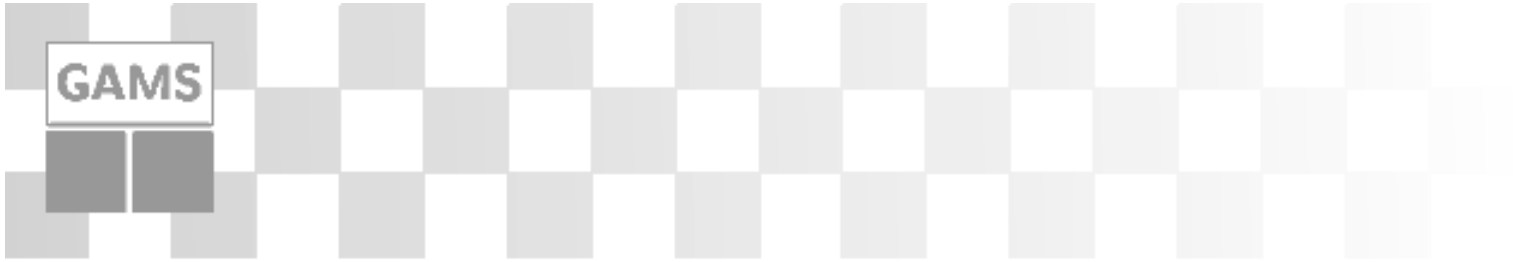


SBB: A New Solver for Mixed Integer Nonlinear Programming

Michael R. Bussieck

GAMS Development Corp.

Arne S. Drud

ARKI Consulting & Development A/S

OR2001, Duisburg

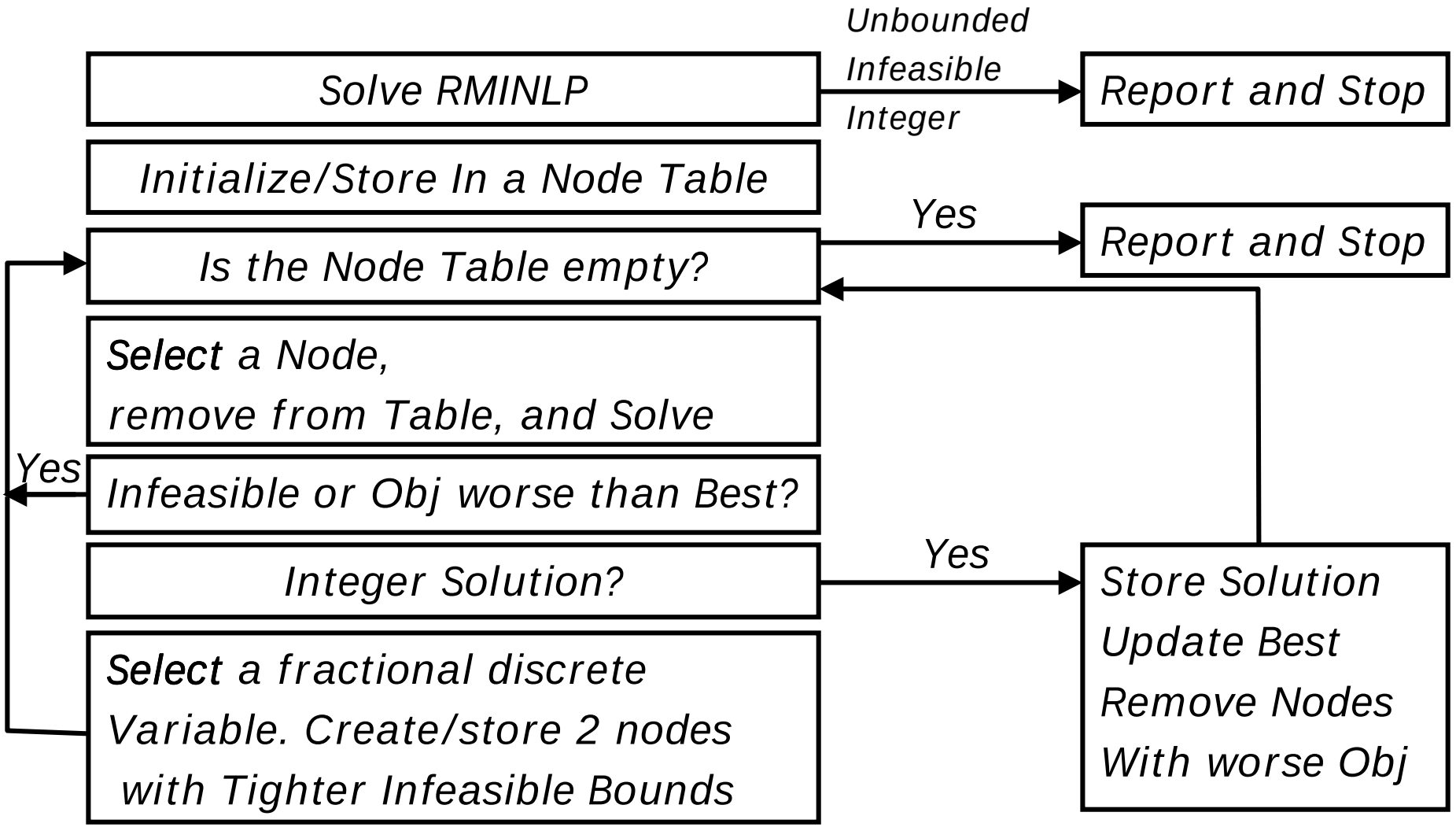
Overview

- SBB = *Simple* Branch & Bound
- Introduction: The MINLP Model
- The B&B Algorithm
- The SBB Design Criteria and Implementation
- Pseudo Cost Variable Selection
- Computational Experiments
- Conclusions and Future Work

The MINLP Model

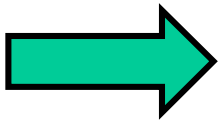
- $$\begin{aligned} &\textit{Min or max } f(x,y) \\ &\textit{s.t.} \quad g(x,y) = b \\ &\qquad l \le x \le u \ \textit{continuous} \\ &\qquad li \le y \le ui \ \textit{integer} \end{aligned}$$
- For convenience we only discuss “Min”
- SBB handles also: SOS1, SOS2, SemiContinuous, and SemiInteger Variables
- Integrality Constraints on y removed: **RMINLP**
- A **Node** is the RMINLP model with tightened bounds on y

The B&B Algorithm



Local/Global Optimization Issues

- Nodes solved with an NLP Local Optimization Algorithm. Potential Problems are:
  - Search Tree can incorrectly be cut off:
    - A node is declared “Infeasible” even though it is only “locally infeasible” and there exist feasible regions.
    - A node is fathomed because the objective for the node is a poor approximation for the global optimum for the node.
    - Problems will occur if the NLP solver **fails**
- RMINLP model is **convex** in both x and y

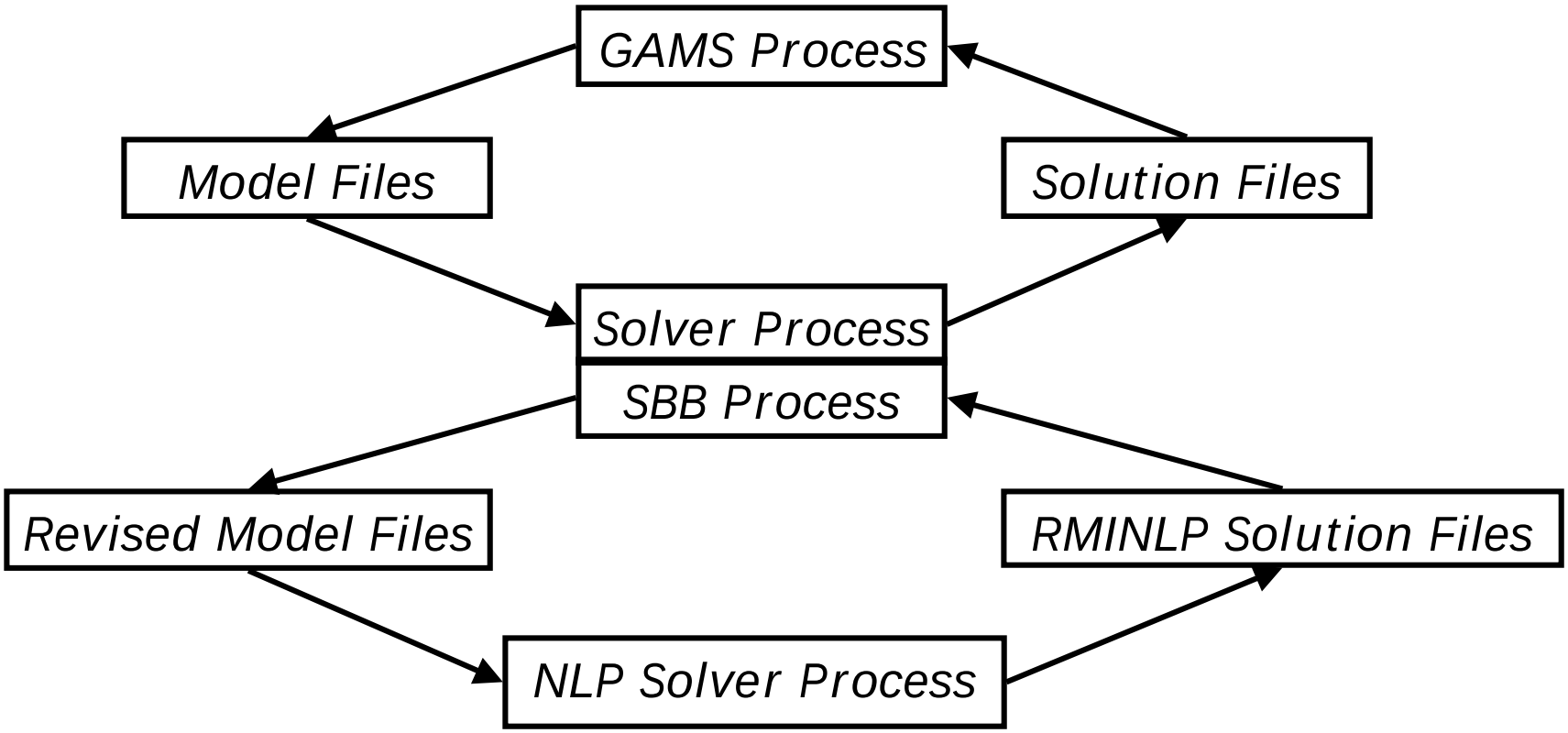


Global Optimum

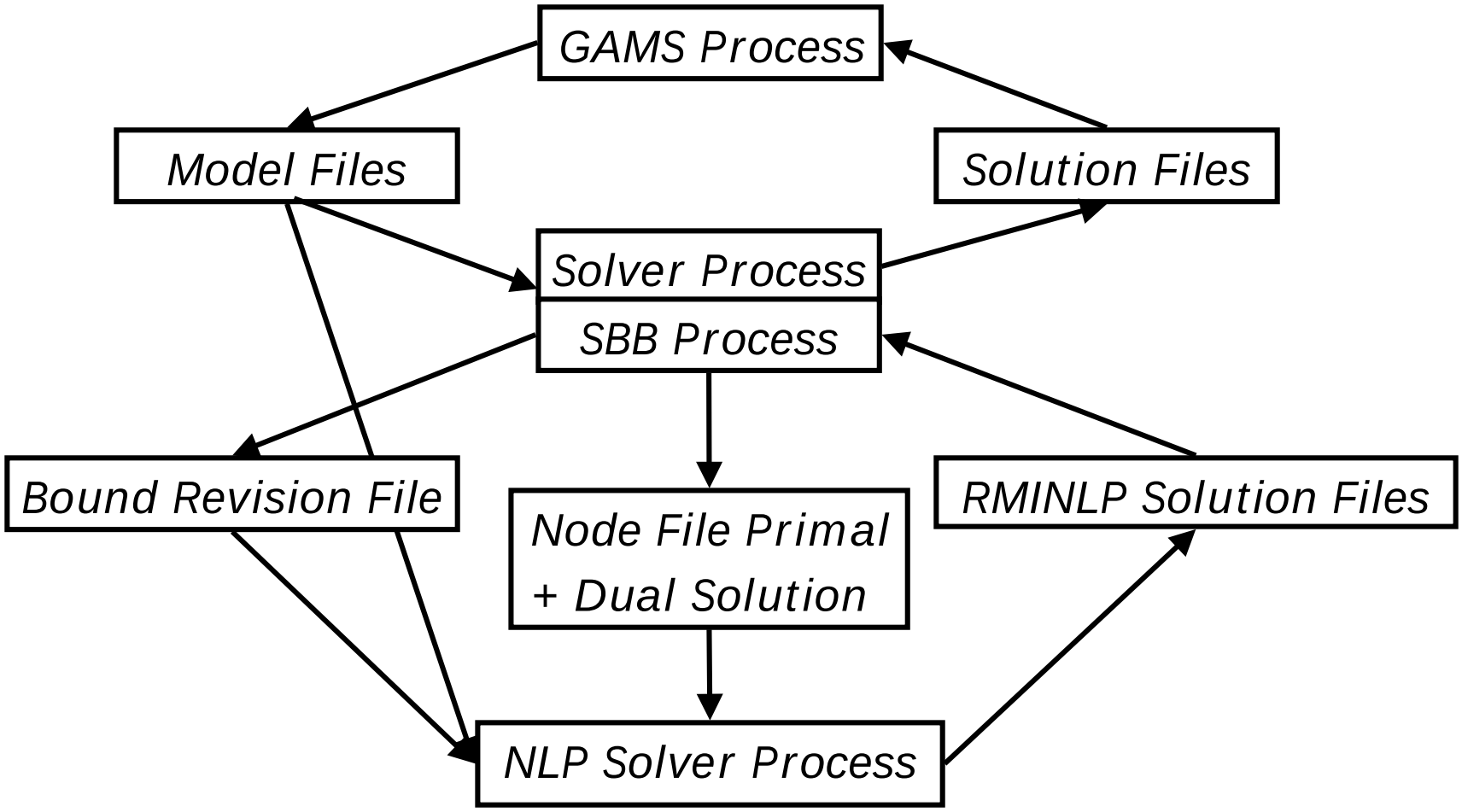
Design Objectives

- Use any existing GAMS NLP solver
  - CONOPT, MINOS, SNOPT
  - PATHNLP, CPLEXSLP
- Handle solver failures
- Help against incorrect infeasibilities
- Report information on non-convexities

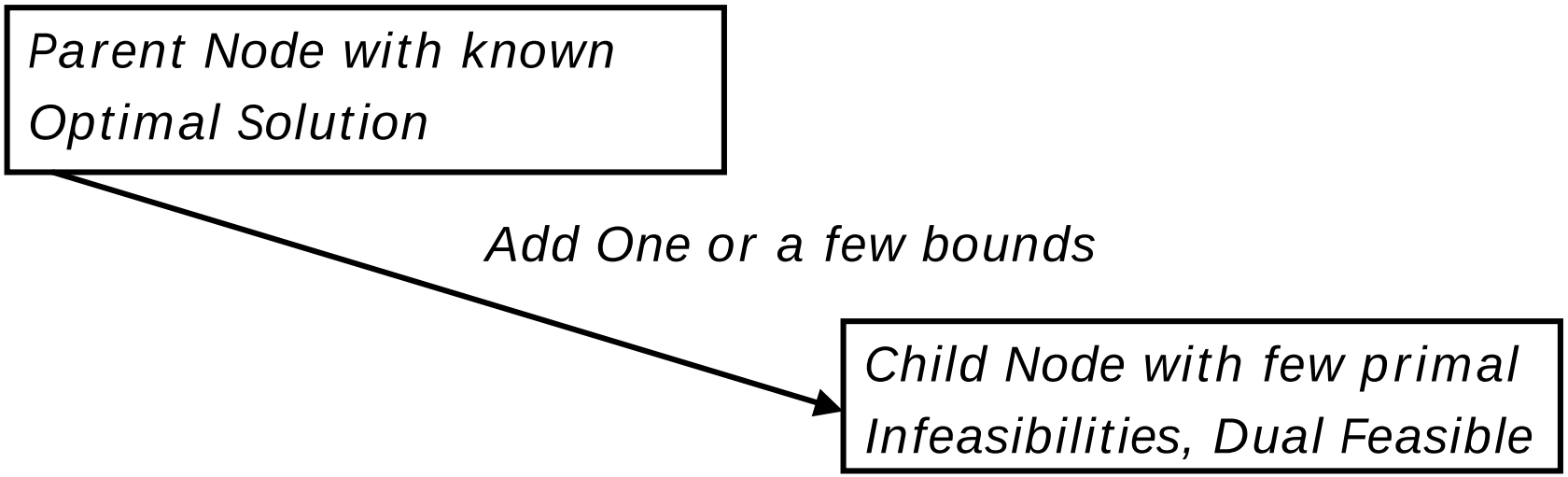
Implementation - Concept



Efficient SBB Implementation



Solving the RMINLP Submodels



Some SBB Options

- ***failseq solver1.opt1 solver2.opt2***
  - Try solver1 with options defined by opt1. If it fails, try solver2 with options defined by opt2, etc. If all fail, **ignore** the node and continue the search
- ***infeasseq level solver1.opt1 solver2.opt2***
  - If a node with depth $\leq$ level is locally infeasible then try solver1 with options defined by opt1, etc.
- ***rootsolver solver.opt***
  - Use a special solver/option pair for the root node
- ***subiter/subres max***
  - Avoid that one node uses all resources

Pseudo Cost Variable Selection

- F set of *binary* variables with fractional value in optimum solution of RMINLP
- |F|>1: Variable selection problem
  - Min/max (integer) infeasibility
  - Pseudo cost (PC)
    - Make important decisions early in the B&B tree
    - Measured by change in the objective of both children nodes
    - Reduces tree size
    - Improves the best bound quickly

PC: Definition and Selection

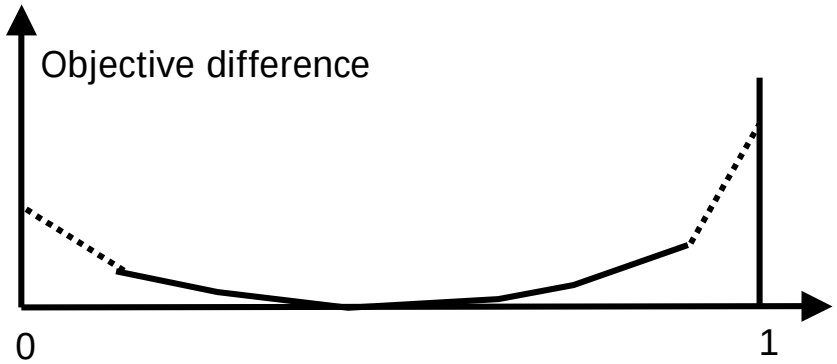
- Definition:
  - *obj** objective in parent node
  - *x** optimum solution in parent node
  - Upbranch: Downbranch:

$$x_i = 1, \overline{c}_i = \frac{\overline{obj} - obj^*}{1 - x_i^*} \qquad x_i = 0, \underline{c}_i = \frac{\underline{obj} - obj^*}{x_i^*}$$

- We don't want $\overline{c}_i = big, \underline{c}_i = tiny$
- $i' = \arg\max_{i \in F}(\min(\overline{c}_i, \underline{c}_i))$

PC: Initialization and Update

- Initialization



- Non linear ???
  - Ask solver (CONOPT) to produce PC by solving 2|F| NLPs internally (without overhead of I/O, presolve, scaling, …)
  - Terminate early if
    - Have large c_i and $\underline{c}_i$
    - Feasible and objdiff tiny

- Update
- *n*th update
  - After solving child node with $x_{i'} = 0$

$$\underline{c}_{i'} = \left(\underline{c}_{i'} + \frac{obj - obj^*}{x_i^*} \right) / n$$

  - After … $x_{i'} = 1$

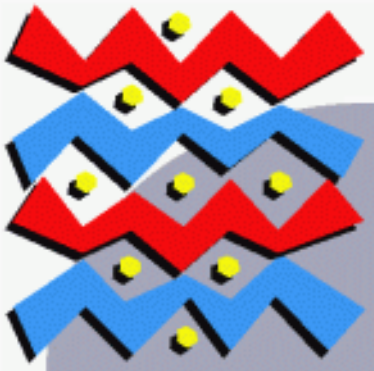
    …

Computational Experiments

- MINLP Solver
  - SBB (Branch & Bound)
  - DICOPT (Outer Approximation)
  - FilterBB (B&B)
  - AlphaECP/mittlp (Extended Cutting Plane)
  - MINOPT/LINGO algorithms
- MINLP Solver (Global Optimization)
  - BARON (Branch and Reduce)
  - OQGRG (Local NLP + Global Search)

Tests

- GAMS/DICOPT
- GAMS/SBB
  - Default, Pseudo Cost Variable Selection
- FilterBB (AMPL Interface)
- Test library MINLPLib
  - 139 MINLP models (existing collections + difficult client models)
  - http://www.gamsworld.org/minlp
  - DICOPT/FilterBB do not support SOS1/2 and semi continuous variables (7 models).
  - 10 minutes time limit (1GHz PC)
  - NLP: CONOPT, MIP: CPLEX



MINLP World

Editorial Board

MINLPLib

MINLP Solvers

MINLP List

Related Links

Search

Contact

Welcome to the MINLP World!

MINLP World is a forum for discussion and dissemination of information about all aspects of Mixed Integer Nonlinear Programming (MINLP).

MINLP models are models that combine combinatorial aspects with nonlinearities. MINLP models are much more difficult than both Mixed Integer Linear Programming (MIP) and Nonlinear Programming (NLP) models.

MINLP is still a new field, and we cannot yet solve all the problems that naturally fall within this area. It is the purpose of this site to bring people that work with MINLP together. We are interested in practical software (MINLP Solvers), testing, comparison, and quality of solvers (MINLPLib), research in both solution methods and in good model formulations, and in improving the communication between people interested in these topics (Related Links and MINLP list).

MINLP World is featured by GAMS World

MINLPLib Model Statistics

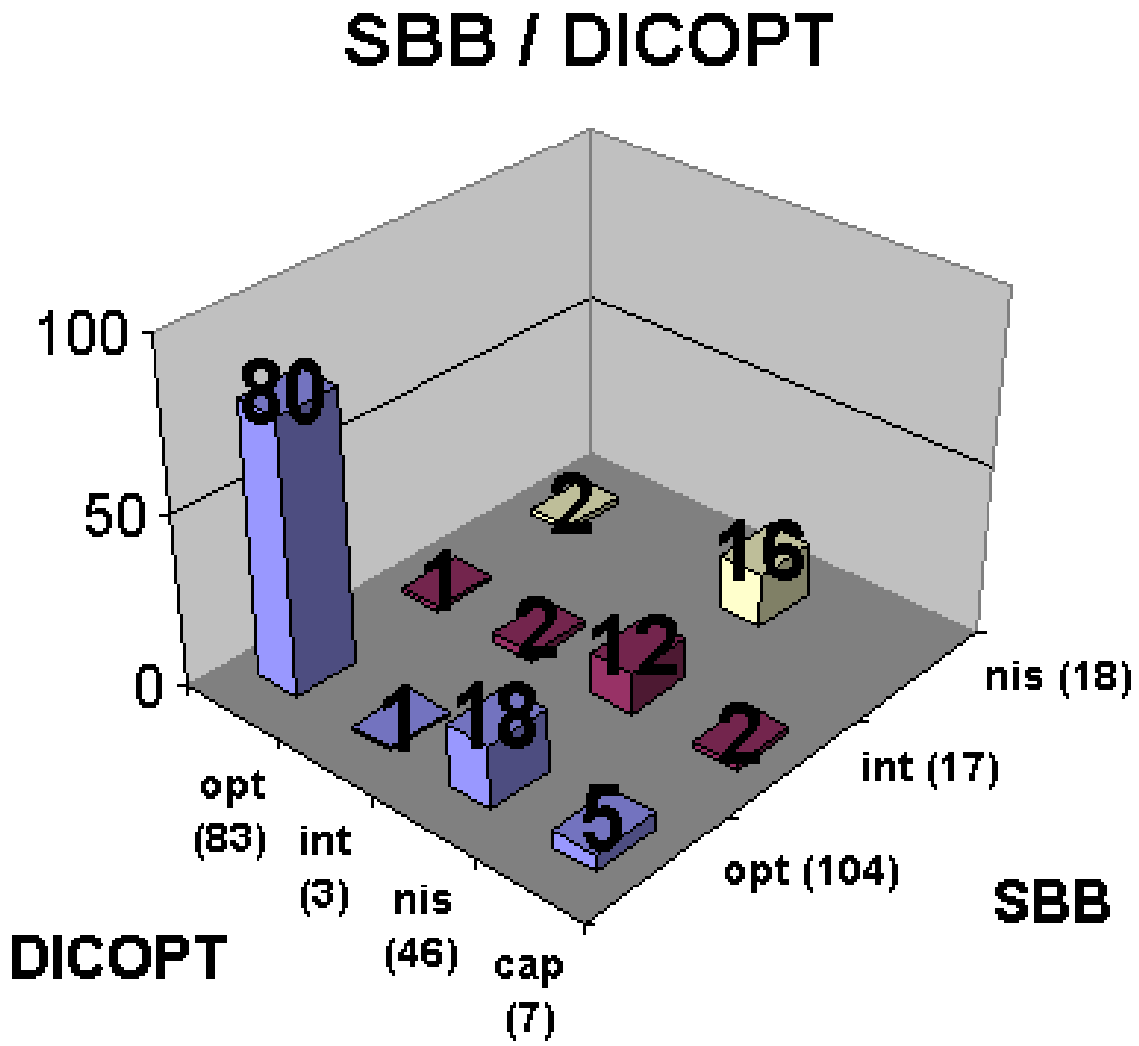
Name	#Eqns	#Vars	#DVars	#NZ	#NNZ	BestInt	at Point
4stufen	99	150	48	319	87	116329.7000	p1
alan	8	9	4	24	3	2.9250	p1
batch	74	47	24	191	22	285506.5000	p1
batchdes	20	20	9	53	10	167427.7000	p1
beuster	115	158	52	398	159	116348.0000	p1
cecil_13	899	841	162	2812	360	-115570.3000	p1
contvar	285	297	87	1281	530	809149.8000	p1
csched1	23	77	63	174	8	-30639.2600	p1
csched2	138	401	308	958	58	-166102.0000	p1
deb10	130	183	11	692	432	209.4278	p1
deb6	508	476	20	2342	1432	201.7393	p1
deb7	898	814	10	4116	2816	116.5846	p1
deb8	898	824	10	4136	2816	116.5846	p1
deb9	918	814	10	4156	2816	116.5846	p1
detf1	6206	4408	400	26610	15400	12.7753	p1
dosemin2d	119	166	32	4379	4080	173.9806	p1
dosemin3d	1003	1047	18	24614	22095	1.4626	p1
elf	39	55	24	178	30	0.1917	p1
eniplac	190	142	24	511	48	-131863.6000	p1
enpro48	215	154	92	742	29	187277.3000	p1
enpro56	192	128	73	651	24	263428.3000	p1
ex1221	6	6	3	17	2	7.6672	p1
ex1222	4	4	1	9	2	1.0765	p1
ex1223	14	12	4	40	17	4.5796	p1
ex1223a	10	8	4	32	9	4.5796	p1
ex1223b	10	8	4	32	17	4.5796	p1
ex1224	8	12	8	31	6	-0.9435	p1
ex1225	11	9	6	27	2	31.0000	p1
ex1226	6	6	3	15	2	-17.0000	p1
ex1233	65	53	12	221	28	155010.7000	p1
ex1243	97	69	16	329	36	83402.5100	p1
ex1244	130	96	23	469	52	82042.9100	p1
ex1252	44	40	15	118	36	128893.7000	p1
ex1252a	35	25	9	94	36	128893.7000	p1

SBB/DICOPT Comparison

Opt = “optimal”

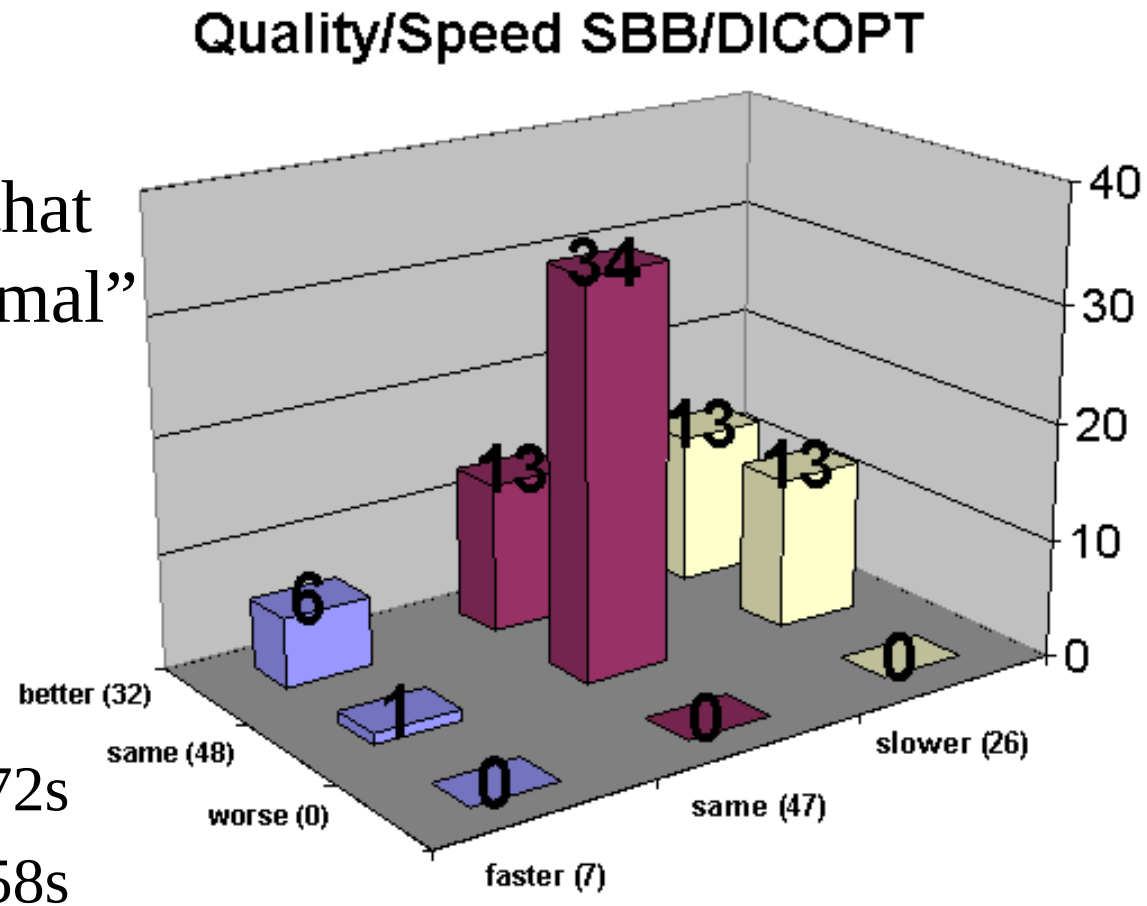
Int = integer solution

Nis = no int solution

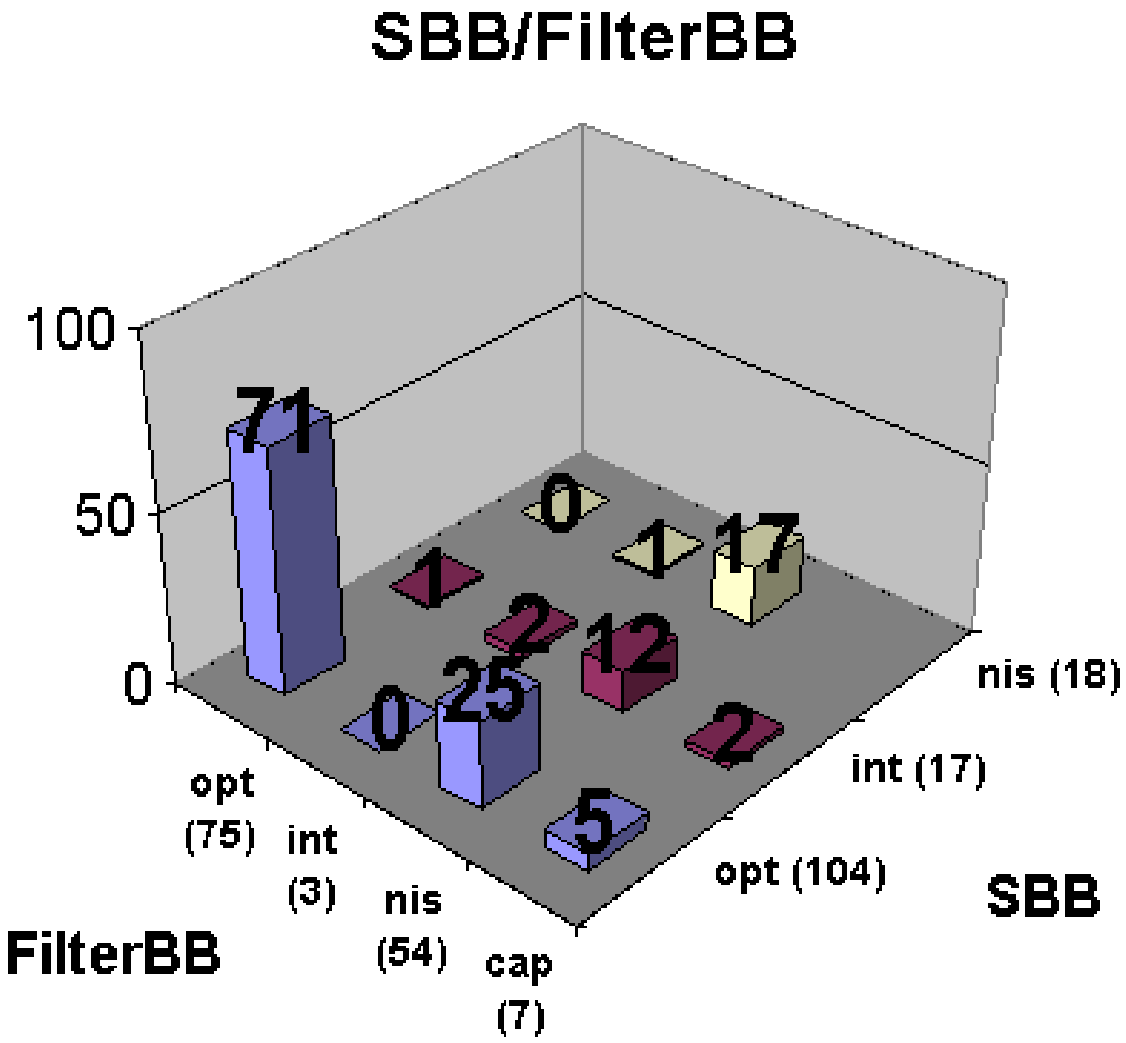


Quality/Speed SBB/DICOPT

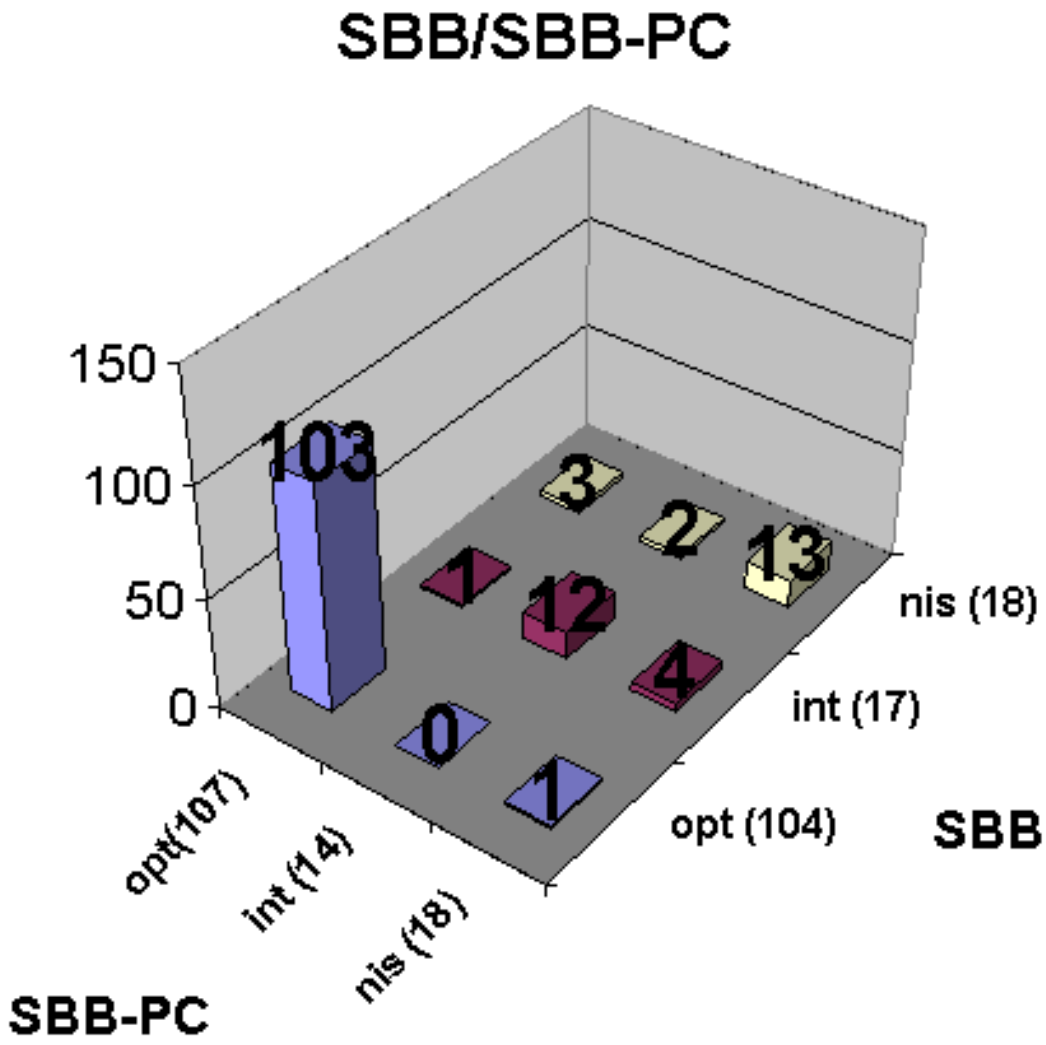
- Test on 80 models that were reported “optimal” in both cases.
- All experiments on 1GHz Linux PC
- Total times
  - SBB: 1572s
  - DICOPT: 258s



SBB/FilterBB Comparison

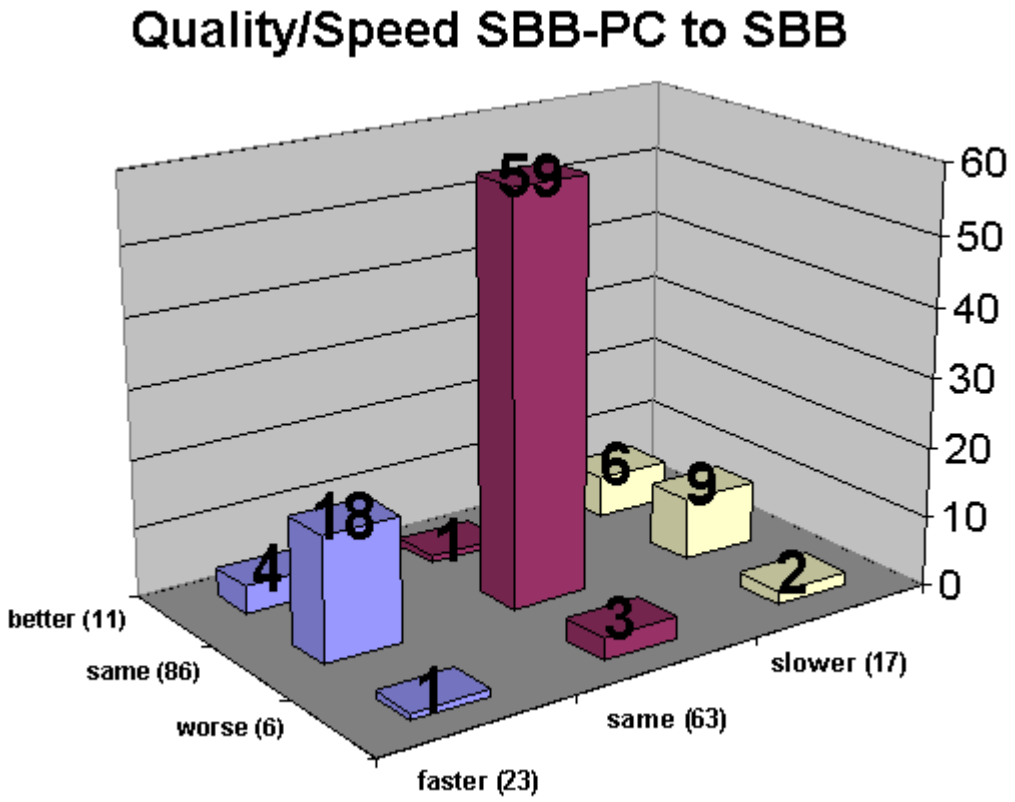


SBB/SBB-PC Comparison



Quality/Speed SBB-PC/SBB

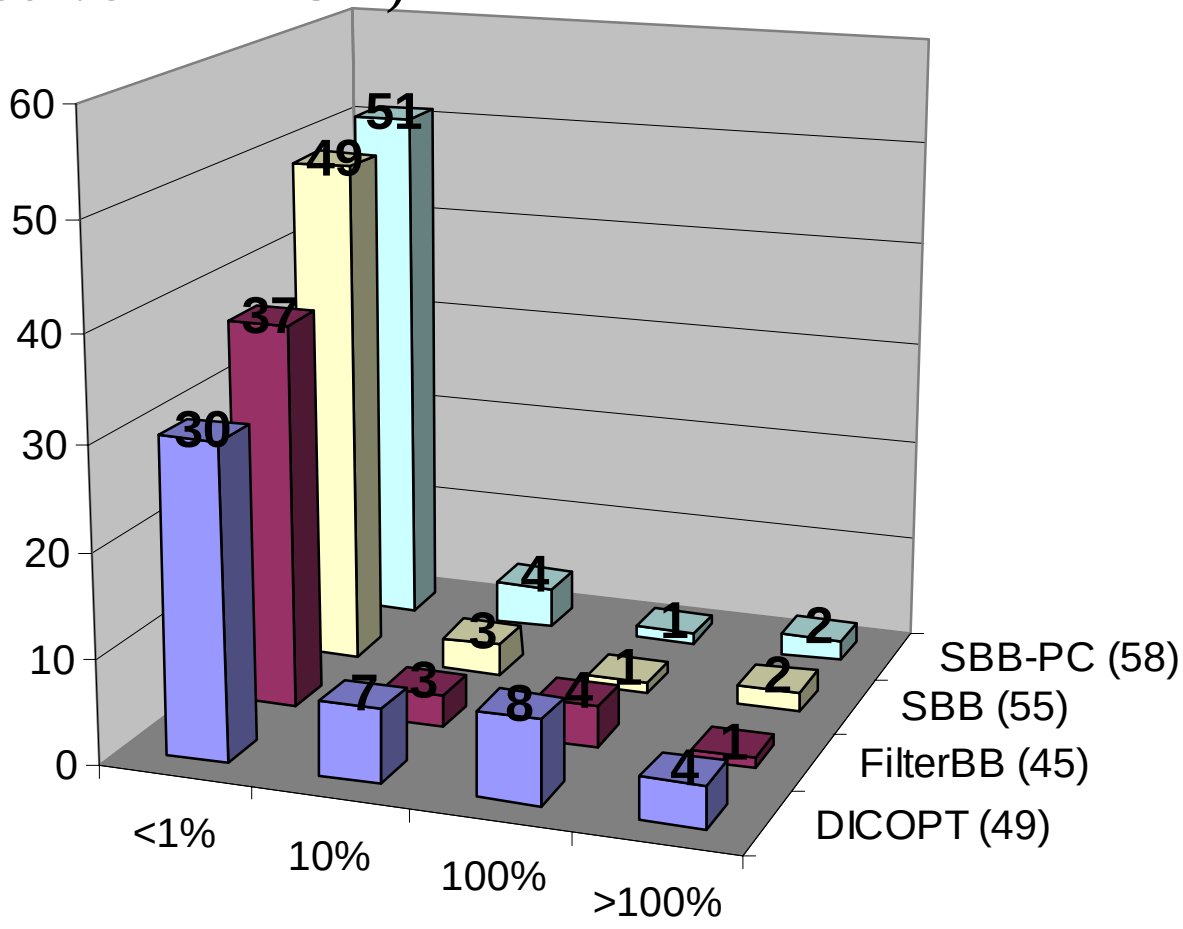
- Test on 103 models that were reported “optimal” in both cases.
- Total times
  - Faster
    - SBB-PC: 1246s
    - SBB: 2187s
  - Slower
    - SBB-PC: 1874s
    - SBB: 599s



Global Solutions???

- Test on 62 models for which the global optimum is known (Global optimization solver BARON)
- Relative gap:

$$rgap = \frac{|obj - obj_{glob}|}{|obj_{glob}|}$$



Future Work/Conclusions

- Today MINLP is at the stage where MIP was long time ago 10-15 years ago
- Model Formulations:
  - Recommendations for good formulations and warnings against bad formulations
  - Preprocessing
- Algorithmic/Theoretical Work:
  - Preprocessing, Probing
  - Cut Generation, …
- Serious MINLP modelers/users need more than one MINLP solver (similar to NLP)